UNIT-3 (continued)

Q.14) Solve the IVP, $\frac{dy}{dx} = \sqrt[3]{x^2+y^2}$, $y(1.1) = 2.02$. Find y at $x = 1.2$ in one step.

Q.15) Solve the IVP, $\frac{dy}{dx} = x^2 + 0.1y^2$, $y(1.3) = 1.02$. Compute y at $x = 1.5$, taking $h = 0.1$.

$y_0 = 2.02 \rightarrow y_1$

$x_0 = 1.1 \quad x_1 = 1.2$

Sol.14) $x_0 = 1.1$, $y_0 = 2.02$, $h = 0.1$

$$f(x,y) = \sqrt[3]{x^2+y^2}$$

$$K_1 = h\, f(x_0, y_0)$$
$$K_1 = 0.1\, f(1.1, 2.02)$$
$$\boxed{K_1 = 0.1742}$$

$$K_2 = h\, f\left(x_0 + \frac{h}{2},\ y_0 + \frac{K_1}{2}\right)$$
$$K_2 = 0.1\, f(1.1 + 0.05,\ 2.02 + 0.0871)$$
$$K_2 = 0.1\, f(1.15,\ 2.1071)$$
$$\boxed{K_2 = 0.1792}$$

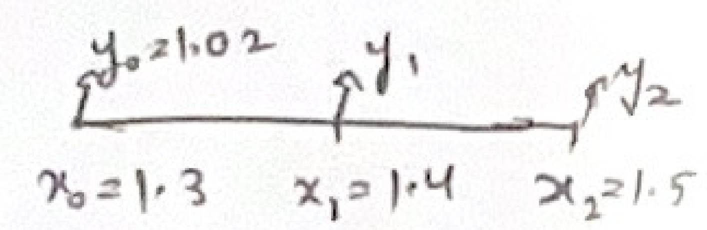
$$K_3 = h\, f\left(x_0 + \frac{h}{2},\ y_0 + \frac{K_2}{2}\right)$$
$$K_3 = 0.1\, f(1.15,\ 2.1096)$$
$$\boxed{K_3 = 0.1793}$$

$$K_4 = h\, f(x_0 + h,\ y_0 + K_3)$$
$$K_4 = 0.1\, f(1.25,\ 2.1993)$$
$$\boxed{K_4 = 0.2505}$$

$$y_1 = y_0 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

$$y_1 = 2.02 + 0.1902$$
$$\boxed{y_1 = 2.2102}$$

Sol. 15 → $x_0 = 1.3$, $y_0 = 1.02$, $h = 0.1$



$y(1.5) = ?$

$f(x,y) = x^2 + 0.1y^2$

$K_1 = h\,f(x_0, y_0)$

$K_1 = 0.1\,f(1.3, 1.02)$

$\boxed{K_1 = 0.1794}$

$K_2 = h\,f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$

$K_2 = 0.1\,f\left(1.3 + \frac{0.1}{2}, 1.02 + \frac{0.1794}{2}\right)$

$K_2 = 0.1\,f(1.35, 1.1097)$

$\boxed{K_2 = 0.1945}$

$K_3 = h\,f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$

$= 0.1\,f(1.35, 1.1172)$

$\boxed{K_3 = 0.1947}$

$K_4 = h\,f(x_0 + h, y_0 + K_3)$

$= 0.1\,f(1.4, 1.2147)$

$\boxed{K_4 = 0.2107}$

$y_1 = y_0 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$y_1 = 1.02 + 0.1947$

$\boxed{y_1 = 1.2147}$

Step II → $x_1 = 1.4$, $y_1 = 1.2147$

$K_1 = h\,f(x_1, y_1)$

$\boxed{K_1 = 0.2107}$

$K_2 = h\,f\left(x_1 + \frac{h}{2}, y_1 + \frac{K_1}{2}\right)$

$= 0.1\,f(1.4 + 0.05, 1.2147 + 0.1053)$

$= 0.1\,f(1.45, 1.32)$

$\boxed{K_2 = 0.2276}$

$K_3 = h\,f\left(x_1 + \frac{h}{2}, y_1 + \frac{K_2}{2}\right)$

$K_3 = 0.1\,f(1.45, 1.3285)$

$\boxed{K_3 = 0.2278}$

$K_4 = h\,f(x_1 + h, y_1 + K_3)$

$K_4 = 0.1\,f(1.5, 1.4425)$

$\boxed{K_4 = 0.2458}$

$y_2 = y_1 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$= 1.2147 + \frac{1}{6}(1.3673)$

$\boxed{y_2 = 1.4425}$

Q 16 → Solve the IVP, $\frac{dy}{dx} = y - x$, $y(0) = 2$. Find $y(0.1)$ and $y(0.2)$ correct to 4 decimal places taking $h = 0.1$

Sol. → $x_0 = 0$, $y_0 = 2$, $h = 0.1$

$f(x,y) = y - x$

$K_1 = h\,f(x_0, y_0)$

$K_1 = 0.1\,(2) = 0.2$

$\boxed{K_1 = 0.2}$

$K_2 = h\,f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$

$= 0.1\,f(0.05, 2.1)$

$\boxed{K_2 = 0.205}$

$$K_3 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$$

$$= 0.1 f(0.05, 2.1025)$$

$$\boxed{K_3 = 0.2052}$$

$$K_4 = h f(x_0 + h, y_0 + K_3)$$

$$= 0.1 f(0.05, 2.1026)$$

$$\boxed{K_4 = 0.2052}$$

$$y_1 = y_0 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

$$y_1 = 2 + 0.2042$$

$$\boxed{y_1 = 2.2042}$$

Step II

$x_1 = 0.1$, $y_1 = 2.2042$, $h = 0.1$

$$f(x, y) = y - x$$

$$K_1 = h f(x_0, y_0)$$

$$\boxed{K_1 = 0.21042}$$

$$K_2 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$$

$$K_2 = h(0.15, 2.3094)$$

$$\boxed{K_2 = 0.2159}$$

$$K_3 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$$

$$\boxed{K_3 = 0.2162}$$

$$K_4 = h f(x_0 + h, y_0 + K_3)$$

$$\boxed{K_4 = 0.2220}$$

$$y_2 = y_1 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

$$y_2 = 2.2042 + 0.2161$$

$$\boxed{y_2 = 2.4203}$$

Boundary-Value Problem (BVP) by finite difference method / approxi

In this method we replace the derivatives occuring in the D.E. as well as in the boundary condition by their approximate finite differences.

To obtain the approximate finite differences we

Expand by taylor series –

$$\boxed{y(x+h) = y(x) + h y'(x) + \frac{h^2}{2!} y''(x) + \frac{h^3}{3!} y'''(x) + \dots} \quad (1)$$

If h is small, neglecting h^2 and their higher power in eqⁿ (1) we get

$$\boxed{y'(x) = \frac{y(x+h) - y(x)}{h} + O(h)} \quad (2)$$

which is known as forward difference approximation of $y'(x)$

Similarly expand in taylor series $y(x-h)$

$$\boxed{y(x-h) = y(x) - h y'(x) + \frac{h^2}{2!} y''(x) - \frac{h^3}{3!} y'''(x) + \dots} \quad (3)$$

If h is small neglecting the power of h^2 & higher power of

$$\boxed{y'(x) = \frac{y(x) - y(x-h)}{h} + O(h)} \quad (4)$$

This is known as ~~finite~~ backward finite difference approxⁿ of first derivative i.e. $y'(x)$.

Subtracting eqⁿ (3) from eqⁿ (1), we get

$$\boxed{y'(x) = \frac{y(x+h) - y(x-h)}{2h} + O(h^2)} \quad (5)$$

which is known as the central finite difference approxⁿ of the first derivative i.e. $y'(x)$

We observe from eq^n (2), eq^n (4) and eq^n (5), the central difference approx^n (5) gives a better approximation.

Adding eq^n (1) & (3), we get

$$\boxed{y''(x) = \frac{y(x+h) - 2y(x) + y(x-h)}{h^2} + O(h^2)} \quad \text{—— (6)}$$

which is known as the finite difference approx^n of the second derivative i.e. $y''(x)$

Now eq^n (2), (4), ~~and~~ (5) & (6) respectively can be expressed in the notation as

$$y_i' = \frac{y_{i+1} - y_i}{h}$$

$$y_i' = \frac{y_i - y_{i-1}}{h}$$

$$y_i' = \frac{y_{i+1} - y_{i-1}}{2h}$$

$$y_i'' = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

where $y_i = y(x_i)$ $\quad i = 0, 1, 2, 3, \dots$

Consider the second order linear differential eq^n

$$y''(x) + p(x)\, y'(x) + q(x)\, y(x) = r(x) \quad \text{—— (7)} \quad x_0 \le x \le x_n$$

where $p(x)$, $q(x)$ & $r(x)$ are known f^n of x. The general boundary condition are given as—

$$\left.\begin{aligned} \alpha y(x_0) + \beta y'(x_0) &= \lambda_1 \\ \gamma y(x_n) + \delta y'(x_n) &= \lambda_2 \end{aligned}\right] \text{—— (8)}$$

where $\alpha, \beta, \gamma, \delta, \lambda_1$ & λ_2 are constants

The finite diff. approx^n of eq^n (7) is

$$\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} + p_i \frac{y_{i+1} - y_i}{h} + q_i y_i = r_i$$

where, $p_i = p(x_i)$ & $q_i = q(x_i)$ and $r_i = r(x_i)$

Multiplying by h^2 on both sides and simplify—

$$y_{i+1} - 2y_i + y_{i-1} + p_i \cdot h\,(y_{i+1} - y_i) + h^2 q_i y_i = h^2 r_i$$

$$\boxed{\left(1 - \frac{h}{2} p_i\right) y_{i-1} + (-2 + h^2 q_i) q_i + \left(1 + \frac{h}{2} p_i\right) y_{i+1} = h^2 r_i} \quad \text{—— (9)}$$

The simplest form of the boundary conditions (8) may be considered when β and δ are zero, i.e. the conditions are of the form

$$y(x_0) = C_1, \quad y(x_n) = C_2$$

where C_1 & C_2 are constants.

Thus from eq^n (9) when i varies from $1, 2, 3, \dots (n-1)$ we get $(n-1)$ system of linear equations with $y_1, y_2, \dots, y_{n-1}$ unknowns

Q17) Solve the boundary value Problem (BVP), $\frac{d^2y}{dx^2} = y$ or —— (1)

$\frac{d^2y}{dx^2} - y = 0$ if the boundary cond^n $y(0) = 0$, $y(2) = 3.627$. Then by finite difference method taking $n = 4$

$y_0 = 0$

$x_0 = 0$ $\quad x_1 = 0.5$ $\quad x_2 = 1$ $\quad x_3 = 1.5$ $\quad x_4 = 2$

$$h = \frac{x_n - x_0}{n} = \frac{2-0}{4} = \frac{1}{2} = 0.5$$

The finite diff. approx^n of the given DE (1)

$$\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} - y_i = 0$$

$$y_{i+1} - (2 + h^2)\, y_i + y_{i-1} = 0 \quad \text{—— (2)} \qquad i = 1, 2, 3$$

Put $i = 1$ in eq^n (2), we get

$$y_2 - (2 + h^2)\, y_1 + y_0 = 0$$

$y_2 - \left(2+\frac{1}{4}\right) y_1 + y_0 = 0$

$y_2 - \frac{9}{4} y_1 = 0$ —— (3)

Put $i = 2$ in eqⁿ (2), we get

$y_3 - \left(\frac{9}{4}\right) y_2 + y_1 = 0$ —— (4)

Put $i = 3$ in eqⁿ (2), we get

$y_4 - \left(\frac{9}{4}\right) y_3 + y_2 = 0$ —— (5)

$3.627 - \frac{9}{4} y_3 + y_2 = 0$

$\frac{9}{4} y_3 - y_2 = 3.627$ —— (5)

Solving eqⁿ (3), (4) & (5)

$y_2 = \frac{9}{4} y_1$

$y_3 - \frac{9}{4}\left(\frac{9}{4}\right) y_1 + y_1 = 0$

$y_3 - \frac{81}{16} y_1 + y_1 = 0$

$\boxed{y_3 = \frac{65}{16} y_1}$

$\frac{9}{4} y_3 - \frac{9}{4} y_1 = 3.627$

$\frac{9}{4}\left(\frac{65}{16} y_1\right) - \frac{9}{4} y_1 = 3.627$

$\left(\frac{585 - 144}{64}\right) y_1 = 3.627$

$\frac{441}{64} y_1 = 3.627$

$\boxed{y_1 = 0.5264}$

$\boxed{y_3 = 2.2847}$

$y_2 = \frac{9}{4} y_1$

$\boxed{y_2 = 1.1843}$

Q18) Solve the BVP, $\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 3y = e^x$ —— (1) subject to the boundary condⁿ $y(0) = 1$, $y(2) = 3.416$ taking the step size, $h = 0.5$

Our aim is to find y_1, y_2 & y_3

$y_0 = 1$, $y_1 = ?$, $y_2 = ?$, $y_3 = ?$, $y_4 = 3$

$x_0 = 0$, $x_1 = 0.5$, $x_2 = 1$, $x_3 = 1.5$, $x_4 = 2$

The finite diff approxⁿ of the given D.E —(1)

$$\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} + 2x_i\left(\frac{y_{i+1} - y_i}{h}\right) - 3y_i = e^{x_i}$$

Multiplying by h^2 on both sides and simplifying we;

$(1 - hx_i) y_{i-1} - (2 + 3h^2) y_i + (1 + hx_i) y_{i+1} = h^2 e^{x_i}$, $i = 1, 2, 3$

Put $i = 1$, we get

$(1 - hx_1) y_0 - (2 + 3h^2) y_1 + (1 + hx_1) y_2 = h^2 e^{x_1}$

$(1 - 0.5 \times 0.5)(1) - [2 + 3(0.5)^2] y_1 + [1 + (0.5)^2] y_2 = (0.5)^2 e^{0.5}$

$0.75 - 2.75 y_1 + 1.25 y_2 = 0.4122$

$1.25 y_2 - 2.75 y_1 = -0.3378$ —— (2)

Put $i = 2$, we get

$(1 - hx_2) y_1 - (2 + 3h^2) y_2 + (1 + hx_2) y_3 = h^2 e^{x_2}$

$(1 - 0.5) y_1 - 2.75 y_2 + (1.5) y_3 = 0.25 e^1$

$0.5 y_1 - 2.75 y_2 + 1.5 y_3 = 0.6796$ —— (3)

Put $i = 3$, we get

$(1 - hx_3) y_2 - (2 + 3h^2) y_3 + (1 + hx_3) y_4 = h^2 e^{x_3}$

$0.25 y_2 - 2.75 y_3 + 1.75 y_4 = 1.1204$

$0.25 y_2 - 2.75 y_3 = -4.8576$ —— (4)

Solving eqⁿ (2), (3) & (4), we get

$\boxed{y_2 = 2.75 y_1 - 0.3378}$

$$0.5y_1 - 2.75\left(\frac{2.75y_1 - 0.3378}{1.25}\right) + 1.5y_3 = 0.6796$$

$$0.5y_1 - (6.05y_1 - 0.74316) + 1.5y_3 = 0.6796$$

$$\boxed{-5.55y_1 + 1.5y_3 = 0.06356}$$

$y_1 = 0.5978$
$y_2 = 0.8513$
$y_3 = 1.8438$

$$0.25\left(\frac{2.75y_1 - 0.3378}{1.25}\right) - 2.75y_3 = -4.8576$$

$$0.55y_1 - 0.06756 - 2.75y_3 = -4.8576$$

$$\boxed{0.55y_1 - 2.75y_3 = -4.79004}$$

$$-10.175y_1 + 2.75y_3 = 0.11653$$
$$+0.55y_1 - 2.75y_3 = -4.79004$$

$$-9.625y_1 = -4.67351$$

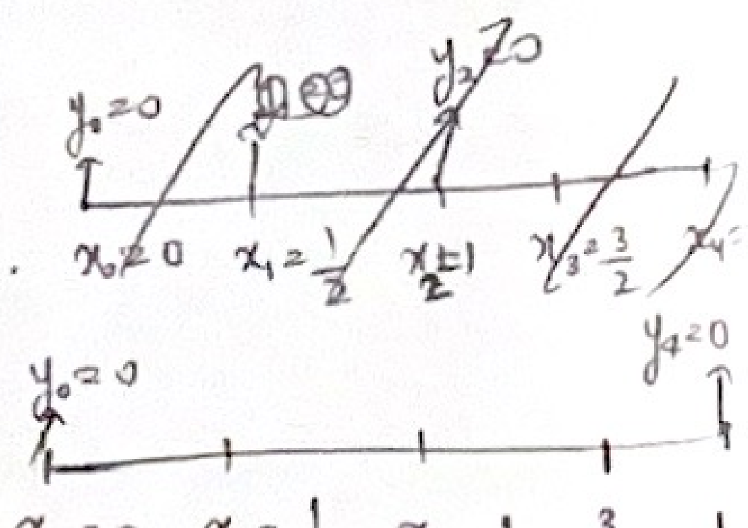
$$\boxed{y_1 = 0.4856}$$

$$\boxed{y_2 = 0.7981}$$

$$\boxed{y_3 = 1.8391}$$

3. Solve the BVP, $y'' - 64y + 10 = 0$ with the boundary cond^n $y(0) = 0$, $y(1) = 0$. Compute $y(0.5)$ choosing–

(i) $n = 2$
(ii) $n = 4$



ii) $n = 4 \Rightarrow$ Divide the interval into 4 parts.

$$h = \frac{x_n - x_0}{n} = \frac{1-0}{4} = \frac{1}{4}$$

$$y'' - 64y + 10 = 0 \quad \text{—— (1)}$$

$$\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} - 64y_i + 10 = 0$$

Multiply by h^2

$$y_{i+1} - 2y_i + y_{i-1} - 64y_ih^2 + 10h^2 = 0$$

~~$y_{i+1} + y_{i-1} + i(64h^2 - 2y_i) + 10h^2 = 0$ —— (2)~~ $\quad i = 1, 2, 3$

~~Put $i = 1$ in eq^n (2), we get~~

~~$y_2 + y_0 +$~~

~~$y_{i+1} + y_{i-1} - 2y_i - 64y_ih^2 + 10h^2 = 0$ —— (2)~~ ~~$i = 1, 2, 3$~~

~~Put $i = 1$ in eq^n (2) we get~~

$$y_{i+1} + y_{i-1} - 2y_i(1 + 32h^2) + 10h^2 = 0 \quad \text{—— (2)} \quad i = 1, 2, 3$$

Put $i = 1$ in eq^n (2) we get

$$y_2 + y_0 - 2y_1\left[1 + 32 \times \left(\frac{1}{4}\right)^2\right] + 10 \times \left[\frac{1}{4}\right]^2 = 0$$

$$y_2 + 0 - 0 + \frac{10}{16} = 0$$

$$\boxed{y_2 = \frac{10}{16}}$$

Put $i = 2$ in eq^n (2) we get

$$y_3 - 2y_2 + y_1 - 64y_2 \times \left(\frac{1}{4}\right)^2 + 10\left(\frac{1}{4}\right)^2 = 0$$

$$y_3 - 2\left(\frac{10}{16}\right) + 0 - 64\left(\frac{10}{16}\right)\left(\frac{1}{16}\right) + \frac{10}{16} = 0$$

$$y_3 = \frac{640}{256} - \frac{10}{16} + \frac{20}{16}$$

$$y_3 = \frac{640 + 160}{256}$$

$$y_3 = \frac{800}{256}$$

$$\boxed{y_3 = \frac{25}{8}}$$

Put $i = 3$ in eq^n (2) we get

$$y_4 + y_2 - 2y_3\left(1 + 32 \times \frac{1}{16}\right) + \frac{10}{16} = 0$$

$$y_4 + \frac{10}{16} - \frac{2 \times 25}{8}(3) + \frac{10}{16} = 0$$

$y_4 + \frac{20}{16} - \frac{75}{4} = 0$

$y_4 = \frac{75}{4} - \frac{20}{16}$

$y_4 = \frac{280}{16}$

$y_4 =$

$y_{i+1} - 2y_i + y_{i-1} - 64 y_i h^2 + 10h^2 = 0$

$y_{i+1} + y_{i-1} - 2y_i(1+32h^2) + 10h^2 = 0$ —— (2) $\quad i = 1, 2, 3$

Put $i=1$ in eq^n (2)

$y_2 + y_0 - 2y_1\left(1 + \frac{32}{16}\right) + \frac{10}{16} = 0$

$y_2 + 0 - 2y_1(3) + \frac{10}{16} = 0$

$\boxed{y_2 - 6y_1 = -\frac{10}{16}}$ —— (3) $\Rightarrow$ $0 - 6y_1 = -\frac{10}{16}$

$\boxed{y_1 = 0.1042}$

Put $i=2$ in eq^n (2)

$y_3 + y_1 - 2y_2\left(1 + \frac{32}{16}\right) + \frac{10}{16} = 0$

$y_3 + y_1 - 6y_2 + \frac{10}{16} = 0$

$\boxed{y_3 + y_1 - 6y_2 = -\frac{10}{16}}$ —— (4)

Put $i=3$ in eq^n (2)

$y_4 - 2y_3 + y_2 - 64 y_3 h^2 + 10h^2 = 0$

$y_4 - 2y_3 + y_2 - \frac{64 y_3}{16} + \frac{10}{16} = 0$

$y_4 - 2y_3 + y_2 - 4y_3 + \frac{10}{16} = 0$

$\boxed{y_4 - 6y_3 + y_2 = -\frac{10}{16}}$ (5)

$y_{i+1} - 2y_i + y_{i-1} - 64 y_i h^2 + 10h^2 = 0$

$y_{i+1} + y_{i-1} - 2y_i(1+32h^2) + 10h^2 = 0$ —— (2) $\quad i = 1, 2, 3$

Put $i=1$ in eq^n (2)

$y_2 + y_0 - 2y_1\left(1 + \frac{32}{16}\right) + \frac{10}{16} = 0$

$y_2 + y_0^{\,0} - 2y_1(3) + \frac{10}{16} = 0$

$\boxed{y_2 - 6y_1 = -\frac{10}{16}}$ —— (3)

Put $i=2$ in eq^n (2)

$y_3 + y_1 - 2y_2(3) + \frac{10}{16} = 0$

$\boxed{y_3 + y_1 - 6y_2 = -\frac{10}{16}}$ —— (4)

Put $i=3$ in eq^n (2)

$y_4 + y_2 - 2y_3(3) + \frac{10}{16} = 0$

$\boxed{y_4 + y_2 - 6y_3 = -\frac{10}{16}}$ —— (5)

Solving eq^n (3), (4) & (5)

$\boxed{y_2 = 6y_1 - \frac{10}{16}}$

$y_3 + y_1 - 6\left(6y_1 - \frac{10}{16}\right) = -\frac{10}{16}$

$y_3 + y_1 - 36y_1 + \frac{60}{16} = -\frac{10}{16}$

$y_3 - 35y_1 = \frac{70}{16}$

$\boxed{y_3 = \frac{70}{16} + 35y_1}$

$y_4 + \left(6y_1 - \frac{10}{16}\right) - 6\left(\frac{70}{16} + 35y_1\right) =$

$y_4^{\,0} + 6y_1 - \frac{10}{16} - \frac{420}{16} - 210y_1 = 7$

$y_4^{\,0} - 204y_1 = \frac{420}{16}$

$\boxed{y_1 = -0.1287}$

$\boxed{y_2 = 0.4538}$

(i) $n=2$ → Divide the interval into 2 subintervals

$y_0=0$, $y_1=?$, $y_2=0$; $x_0=0$, $x_1=0.5$, $x_2=1$

$$h=\frac{x_n-x_0}{n}=\frac{1-0}{2}=0.5$$

$$y''-64y+10=0$$

$$\frac{y_{i+1}-2y_i+y_{i-1}}{h^2}-64y_i+10=0$$

$$y_{i+1}-2y_i+y_{i-1}-64y_i(h^2)+10h^2=0$$

$$y_{i+1}+y_{i-1}-2y_i(1+32h^2)+10h^2=0 \quad \text{——— (2)} \qquad i=1,2,3$$

Put $i=1$ in eq^n (2)

$$y_2+y_0-2y_1\left(1+\frac{32}{4}\right)+\frac{10}{4}=0$$

$$\cancel{y_2}+\cancel{y_0}-18y_1+\frac{5}{2}=0$$

$$-18y_1=-\frac{5}{2}$$

$$\boxed{y_1=0.1389}$$

Q20 Solve the BVP, $y''-xy'+2y=\log(1+x)$, with cond^n $y(0)=1$ and $y(1)=0$, $h=0.25$.

$y_0=1$, $y_1=?$, $y_2=?$, $y_3=?$, $y_4=0$; $x_0=0$, $x_1=0.25$, $x_2=0.5$, $x_3=0.75$, $x_4=1$

REMARK So far we have considered the simplest form of the boundary condition, let us modify these conditions.

Q Solve the BVP, $\frac{d^2y}{dx^2}-y=0$ ——— (1) with the boundary condition $y'(0)=0$ ——— (2) and $y(1)=1$ by finite difference method taking $n=2$.

$y_2=1$; $x_0=0$, $x_1=0.5$, $x_2=1$

The finite approximation of eq^n (1)

$$\frac{y_{i+1}-2y_i+y_{i-1}}{h^2}-y_i=0$$

Multiplying by h^2 & simplifying we get

$$y_{i+1}-(2+h^2)y_i+y_{i-1}=0 \quad \text{——— (3)}$$

Put $i=0$ in eq^n (3)

$$y_1-2.25y_0+y_{-1}=0 \quad \text{——— (4)}$$

The finite diff. approx^n of eq^n (2) is —

$$\left[\frac{y_{i+1}-y_{i-1}}{2h}\right]_{i=0}=0$$

$$\therefore\ y_1-y_{-1}=0$$

$$\Rightarrow \boxed{y_1=y_{-1}}$$

Put value of y_{-1} in (4)

$$y_1-2.25y_0+y_1=0$$

$$\boxed{2y_1=2.25y_0} \quad \text{——— (5)}$$

Put $i=1$ in eq^n (3) we get

$y_2 - (2+h^2)y_1 + y_0 = 0$

$y_2 - 2.25y_1 + y_0 = 0$

$1 - 2.25y_1 + y_0 = 0$

$\boxed{2.25y_1 - y_0 = 1}$ (6)

$(2.25y_1 - y_0 = 1) \times 2.25$

$2y_1 - 2.25y_0 = 0$

$5.0625y_1 - \cancel{2.25y_0} = 2.25$

$2y_1 - \cancel{2.25y_0} = 0$

$\quad - \quad + \quad -$

$3.0625y_1 = 2.25$

$\boxed{y_1 = 0.7347}$

$\boxed{y_0 = 0.6531}$

Q. Solve the BVP, $\frac{d^2y}{dx^2} - y = 0$ (1), take boundary cond^n $y(0)=0$, $y'(1)=1$ (2) taking $n=2$

$y_0=0$

$x_0=0 \quad x_1=0.5 \quad x_2=1$

The finite approx^n of eq^n

$\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} - y_i = 0$

Multiplying by h^2 & simplifying we get

$y_{i+1} - (2+h^2)y_i + y_{i-1} = 0$ —— (3)

Put $i=2$ in (3)

$y_3 - 2.25y_2 + y_1 = 0$

$\cancel{y_2 - (2+0.25)y_1 + y_0 = 0}$

$\cancel{y_2 - 2.25y_1 + y_0 = 0}$

Put $i=1$ in (3)

$y_2 - 2.25y_1 + y_0 = 0$

$y_2 - 2.25y_1 = 0$

$y_1 + y_{-1} = 0$

The finite diff. approx. of eq^n (2)

$\cancel{\frac{y_{i+1} - y_{i-1}}{2h}} \quad \cancel{y(x_i) - y_i}$

$\cancel{\frac{y_i - y_{i-1}}{h} = 1}$

$\cancel{y_2 = 2.25y_1}$

$\left[\frac{y_{i+1} - y_{i-1}}{2h}\right]_{i=2} = 1$

$\frac{y_3 - y_1}{2h} = 1$

$y_3 - y_1 = 2h$

$\boxed{y_3 - y_1 = 1}$

$y_3 = 1 + y_1$

$1 + y_1 - 2.25y_2 + y_1 = 0$

$2y_1 - 2.25y_2 + 1 = 0$

$(y_2 - 2.25y_1 = 0) \times 2.25$

$\cancel{2.25y_2 - 5.0625y_1 = 0}$

$\cancel{-2.25y_2 + y_1 = -1}$

$\cancel{-4.0625y_1 = -1}$

$2y_1 - \cancel{2.25y_2} + 1 = 0$

$-5.0625y_1 + \cancel{2.25y_2} = 0$

$-3.0625y_1 = -1$

$\boxed{y_1 = 0.3267}$

$y_2 = 2.25y_1$

$\boxed{y_2 = 0.7351}$

$y_1 = 0.3265$

$y_2 = 0.7347$

Q Solve the BVP, $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + xy = e^x$, $\cancel{y(0)}$ $y(0) - 2y'(0) = 1.1$ —— (1)

$y(0.9) = 2.6$ —— (2) , $h = 0.3$ by finite diff. approx^n method

The finite diff. approx^n of the given D.E (1)

$$\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} - 2\left(\frac{y_{i+1} - y_{i-1}}{2h}\right) + x_i y_i = e^{x_i}$$

Multiplying by h^2 on both sides and collecting the terms, we get

$(1+h)\, y_{i-1} + (-2 + h^2 x_i) y_i + (1-h) y_{i+1} = h^2 e^{x_i}$, $i = 0, 1, 2, \ldots$ —— (3)

The finite diff. approx^n of its boundary cond^n (2) is

$$\left[y_i - 2\left(\frac{y_{i+1} - y_{i-1}}{2h}\right)\right]_{i=0} = 1.1$$

y_0 y_1 y_2 $y_3 = 2.6$

$x_0 = 0$ $x_1 = 0.3$ $x_2 = 0.6$ $x_3 = 0.9$

$h y_0 - y_1 + y_{-1} = 1.1h$

$y_{-1} = 1.1h - hy_0 + y_1$

Put $i = 0$ in eq^n (3) and simplify, we get

$-2.39 y_0 + 2y_1 = 0.339$ —— (4)

Put $i = 1$ in eq^n (3) and simplify, we get

$1.3 y_0 - 1.973 y_1 + 0.7 y_2 = 0.1215$ —— (5)

Put $i = 2$ in eq^n (3) and simplify, we get

$1.3 y_1 - 1.946 y_2 = 0.1640 - 0.7 y_3$ —— (6)

Solving the eq (4), (5), & (6), we get

$y_0 = 1.4618$

$y_1 = 1.5774$

$y_2 = 1.9047$

Q Solve the BVP $\frac{d^2y}{dx^2} - \frac{2dy}{dx} + xy = e^x$ —— (1), with the boundary condition, $y(0) = 1$, $y(0.9) + 3y'(0.9) = 3$ —— (2) $0 \le x \le 0.9$

$y_0 = 1$

$x_0 = 0$ $x_1 = 0.3$ $x_2 = 0.6$ $x_3 = 0.9$

$h = 0.3$

As in the previous ques. the finite diff. approx^n of the eq^n (1) is

$(1+h) y_{i-1} + (-2 + h^2 x_i) y_i + (1-h) y_{i+1} = h^2 e^{x_i}$ —— (3)

and the finite diff. approx^n of its boundary cond^n is

$$\left[y_i + 3\left(\frac{y_{i+1} - y_{i-1}}{2h}\right)\right]_{i=3} = 3$$

$y_4 = y_2 - 0.2 y_3 + 0.6$ —— (4)

Put $i = \cancel{0}1, 2, 3$ in eq^n (3) and using the value of y_4 from eq^n we get the 3 eq^n, solving these 3 eq^n we get the value

$i=1$ $1.3 y_0 + (-2 + 0.09 \times 0.3) y_1 + (0.7) y_2 = 0.09 e^{0.3}$

$1.3 y_0 - 1.973 y_1 + 0.7 y_2 = 0.1215$

$-1.973 y_1 + 0.7 y_2 = -1.1785$ —— (5)

$i=2$ $1.3 y_1 - 1.946 y_2 + 0.7 y_3 = 0.164$ —— (6)

$i=3$ $1.3 y_2 - 1.919 y_3 + 0.7 y_4 = 0.2214$ —— (7)

$1.3 y_2 - 1.919 y_3 + 0.7 (y_2 - 0.2 y_3 + 0.6) = 0.2214$

$2y_2 - 2.059 y_3 + 0.42 = 0.2214$

$(2y_2 - 2.059 y_3 = -0.1986$ —— (7) $) \times 1.4848$

$$1.3\left(\frac{0.7y_2+1.1785}{1.8973}\right)-1.946y_2+0.7y_3=0.164$$

$$0.4612y_2+0.7765-1.946y_2+0.7y_3=0.164$$

$$(-1.4848y_2+0.7y_3=-0.6125)\times 2$$

$$-2.9696y_2+1.4y_3=-1.225$$

$$2.9696y_2-3.057y_3=0.2949$$

$$-1.657y_3=-0.9301$$

$$\boxed{y_3=0.5613}$$

$y_1=0.8971$

$y_2=0.8449$

$y_3=0.9172$

$$2y_2=2.059(0.5613)-0.1986$$

$$2y_2=0.9571$$

$$\boxed{y_2=0.4786}$$

$$y_1=\cancel{0.7y_2}\ \frac{0.7(0.4786)+1.1785}{1.973}$$

$$=\frac{0.3350+1.1785}{1.973}$$

$$\boxed{y_1=1.5555}$$